Indoor positioning using Extended Kalman Filter

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# Introduction

This report summarizes the short investigation of the indoor positioning problem faced by Tracxpoint and suggests a possible solution; the Extended Kalman Filter . The chosen technology is the Ultra Wideband (UWB) and the device is the almost industry standard Decawave’s DW1000.

# Trilateration [1]

The basic information provided by the DW1000 is the Time Of Flight (TOF) from a given tag to an anchor. This time of flight is assumed to reflect the distance according to:

|  |  |  |
| --- | --- | --- |
|  |  | ‎2.1 |

Where; , is the speed of light, and is the anchor’s index.

The basic positioning derivation technique is the Trilateration; TOF between a tag and 3 or more anchors allows to intersect 3 imaginary spheres having each a radius . In fact, since the heights of both tag and anchor are known and constant, we may conduct all the calculation in a 2-dimensional plane (Pythagorean triplet);

|  |  |  |
| --- | --- | --- |
|  |  | ‎2.2 |

Where;

|  |  |  |
| --- | --- | --- |
|  |  | ‎2.3 |

An illustration of the intersection is brought in Figure 1;

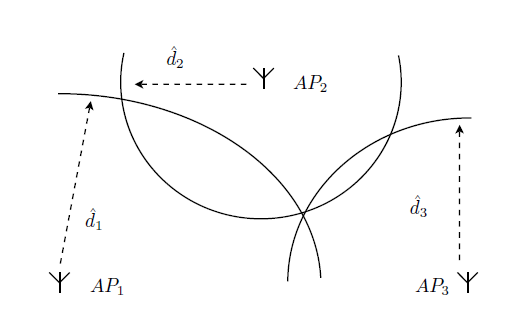


Figure 1: Trilateration

The Trilateration is a “memoryless” technique, as it uses only the current observation to derive the location. Another memoryless technique exists, Multilateration, but is much less used for the reasons mentioned in [2].

# Extended Kalman Filter [3]

## Problem formulation

The Kalman Filter has been widely used in motion tracking problems, whose observations are noisy. In our case, the estimated parameter is the current position and velocity of an object, the state vector, and the observations are the distances (or the square distances) to the anchors. In general, the state and the observation are vectors. The Kalman estimator regards the state and its observation as a Hidden Markovian Random Process (HMM):

…

Figure 2: HMM process model

Hence, known algebraic and probabilistic relationships between current state and observation to the previous.

Probabilistic relationship;

|  |  |  |
| --- | --- | --- |
|  |  | ‎3.1 |

The physical model assumed by the Kalman filter is expressed by the algebraic relationship:

|  |  |  |
| --- | --- | --- |
| The State equation |  | ‎3.2 |

|  |  |  |
| --- | --- | --- |
| The Measurement equation |  | ‎3.3 |

Where;

* is the current step’s input to the system
* is the previous step’s added noise; the state equation noise. It is assumed Gaussian;
* is the current step’s added noise; the measurement equation noise. It is assumed Gaussian;

The functions are not necessarily linear, thus unfit to be treated by the Basic Kalman filter.

## Estimation

Let’s define the:

* Apriori state estimate as the one based on , its respective error, and error covariance matrix;

The apriori state estimator has a significance of prediction as it relies on past state and past measurement

* Posteriori state estimate as the one based on , its respective error, and error covariance matrix;
* the innovation process or residual

which represents the gap between the predicted measurement and the actual one.

We suggest the posteriori state estimator as the following linear combination:

|  |  |  |
| --- | --- | --- |
|  |  | ‎3.4 |

Both apriori estimator and current measurement participate naturally, and their respective contribution is adjusted at every step by the blending factor . The heart of the optimality resides in the value of the blending factor; it is such that the trace of which is the sum of the variances of the terms of , is minimized. We can also see that the bigger the residual the greater its potential to have an impact on the posteriori estimator; the more information it adds to the existing one. The actual impact depends also on . The posteriori estimator can may be regarded as the correction of the apriori estimator.

The apriori is calculated in his turn from:

|  |  |  |
| --- | --- | --- |
|  |  | ‎3.5 |

Where is the posteriori estimator.

The pair ‎3.4 and ‎3.5 compose together a “prediction-correction” sequential pair, in which ‎3.7 precedes ‎3.4.

We can show that small measurement noise and /or large state noise lead to large ; measurement is reliable comparing to state, and vice versa.

## Sequential EKF algorithm

The ‎3.4 and ‎3.5 equations together with the derivation of optimal lead to the following sequential algorithm:

* **Prediction/ Time update:**

|  |  |  |
| --- | --- | --- |
| Apriori state estimate |  | ‎3.6 |

|  |  |  |
| --- | --- | --- |
| Apriori error covariance estimate |  | ‎3.7 |

* **Correction/ Measurement update:**

|  |  |  |  |
| --- | --- | --- | --- |
| Blending factor |  | ‎3.8 | |
| Posteriori state estimate |  | ‎3.9 |
| Posteriori error covariance estimate |  | ‎3.10 |
|  |  |  |

In which appear the Jacobians:

# Application of EKF to Indoor Positioning [4]

## State equation

In the indoor positioning case, the state, is a vector;

|  |  |  |
| --- | --- | --- |
|  |  | ‎4.1 |

Where are the 2D position and velocity respectively.

### Known acceleration

The state equation describes an accelerated movement, where the 2D acceleration is known in principle;

|  |  |  |
| --- | --- | --- |
|  |  | ‎4.2 |
|  |  |  |

And is thus linear:

The acceleration is treated as an input and is supposed to be provided by an acceleration sensor (accelerometer). The noise is modeling the accelerometer inaccuracy and is supposed to be white;

Hence;

|  |  |  |
| --- | --- | --- |
|  |  | ‎4.3 |

The value of and should be provided by the accelerometer device.

### Unknown acceleration

When the acceleration is unknown, we can regard it as a white random process. The state equation in this case is;

Whose noise vector is:

The noise covariance matrix is:

|  |  |  |
| --- | --- | --- |
|  |  | ‎4.4 |

and are and respectively (). A good choice would be the maximum possible acceleration on each of the axes. The null terms in the matrix are the result of lack of correlation between and .

## Measurement equation

The observation vector;

|  |  |  |
| --- | --- | --- |
|  |  | ‎4.5 |

And the observation equation is;

|  |  |  |
| --- | --- | --- |
|  |  | ‎4.6 |

Which is obviously nonlinear.

The noise is modeling the TOF measurement inaccuracy and is presumed white;

|  |  |  |
| --- | --- | --- |
|  |  | ‎4.7 |

## Jacobians

Where , and

Where and

Where , and

Where , and

## Initial state [4]

The EKF is a sequential algorithm, and it is therefore useful to have an initial state that is as accurate as possible. One option is to solve the nonlinear system ‎2.3. It is also possible to formulate a linear least squares problem (true for all , namely for );

Which becomes;

And in matrix form;

Solving for , we may derive to give (assuming initial velocities are zero):

|  |  |  |
| --- | --- | --- |
|  |  | ‎4.8 |

# Simulations

We have a recording of a moving tag along a known grid ( between each grid); ranges from 3 anchors sampled at a interval.

We have estimated the track using Trilateration and EKF. The EKF supposed unknown acceleration. The EKF’s model tunable parameters are;

* The measurement noise: (measured in units of )
* The state noise: (measured in units of )

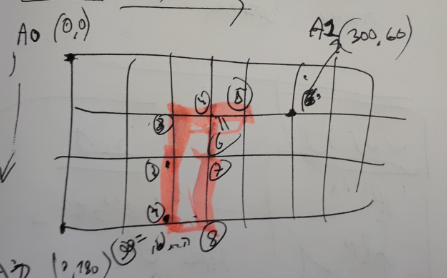
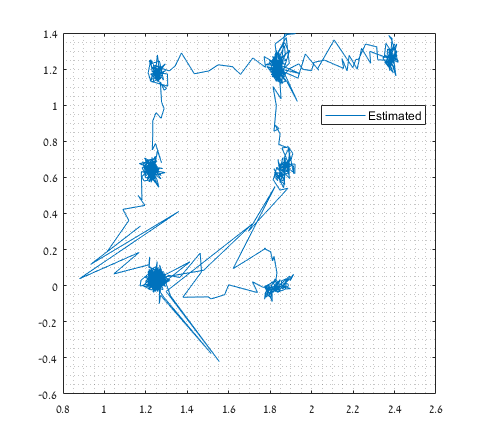


Figure 3: Ground Truth. Grid=

Figure 4: Trilateration

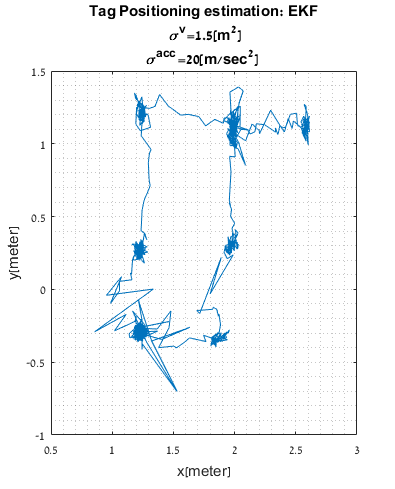
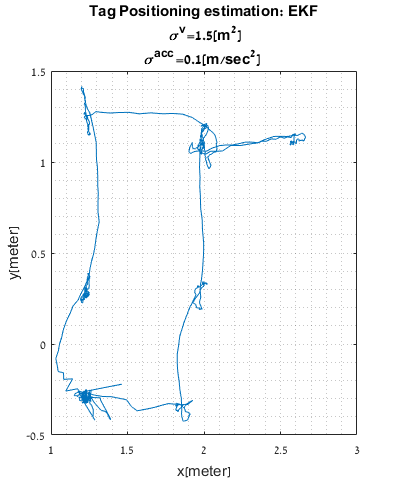


Figure 5: EKF- performance versus state noises/acceleration. Large state noise (left), small state noise (right)

We may notice the large fluctuations in the trilateration case comparing to the EKF case. Also, we detect the resemblance of the EKF with large state noise to the trilateration. This makes sense, as the noisier the previous state the less we rely on it on estimating the current state and the more on the current measurement; which is exactly what the trilateration does

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|  |  |
| --- | --- |
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